

# Bridging the gap by shaking superfluid matter

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In cold compact stars, Cooper pairing between fermions in dense matter leads to the formation of a gap in their excitation spectrum and typically exponentially suppresses transport properties. However, we show here that weak Urca reactions become strongly enhanced and approach their ungapped level when the star undergoes density oscillations of sufficiently large amplitude. We study both the neutrino emissivity and the bulk viscosity due to direct Urca processes in hadronic, hyperonic and quark matter and discuss different superfluid and superconducting pairing patterns.

The dense matter in a compact star has certain transport properties, such as bulk viscosity and neutrino emissivity, that are dominated by beta (weak interaction) equilibration processes. However, this equilibration rate is expected to be exponentially suppressed as the star cools below the critical temperature for superconductivity/superfluidity, and Cooper pairing produces a gap in the relevant fermion excitation spectrum [1, 2]. This suppression is relevant both in hadronic matter, where the critical temperature for proton superconductivity and neutron superfluidity is in the range  $T_c = 10^8 - 10^{10}$  K [3] and in exotic phases such as quark matter, where critical temperatures as high as  $T_c \simeq 3 - 5 \times 10^{11}$  K if flavor antisymmetric pairing channels are favored, and as low as  $T_c \simeq 10^7$  K when flavor singlet pairing is favored [4].

In this letter we show that the exponential suppression operates only at small amplitude and may be overcome in realistic situations. It is already known that beta-equilibration rates in normal matter are enhanced by suprathermal processes at high amplitude when equilibration is driven by higher order terms in the chemical potential imbalance [5–7]. Here we report on a similar but more dramatic phenomenon in superfluid/superconducting phases of matter, arising from a threshold-like behavior with a rapid increase in available phase space when the typical energy in the equilibration processes approaches the gap.

The mechanism we discuss is generic and can be expected to operate in all situations where large perturbations drive the system out of beta-equilibrium. One possible application are unstable oscillations of rotating compact stars, like f- or r-modes [8]. These modes are unstable due to gravitational wave emission and their amplitude grows exponentially until they are saturated by non-linear coupling to damped daughter modes [9] or damped by the supra-thermal bulk viscosity [10]. Note, that even at amplitudes well below the level where bulk viscosity alone would saturate such modes, suprathermal effects can noticeably affect the thermal evolution of the star, via viscous heating and suprathermally enhanced neutrino emissivity [6]. Another class of scenarios stems from large amplitude oscillations caused by singular events, like star quakes [11] or tidal forces preceding

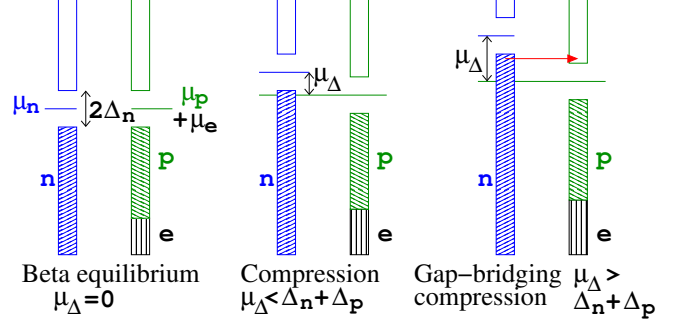


Figure 1: Schematic illustration of the opening of phase space for Urca interactions in gapped nuclear matter when the deviation from beta equilibrium  $\mu_\Delta = \mu_n - \mu_p - \mu_e$  becomes sufficiently large relative to the pairing gaps  $\Delta_n, \Delta_p$ .

neutron star mergers [12]. In this case the suprathermal bulk viscosity should control the amplitude of the oscillation and correspondingly the emitted radiation.

Here we will study the effect of large amplitude oscillations in the case of various *gapped* phases of dense matter. When, as in nuclear matter, Cooper pairs contain two particles of the same flavor, density oscillations cannot lead to pair breaking. (Thermal pair-breaking has been used [13, 14] to explain the rapid cooling observed in Cas A [15].) Nevertheless, oscillations displace the gapped Fermi seas, and at sufficiently large amplitude can *bridge the gap*, opening phase space for beta equilibration processes as illustrated in fig. 1. As we will show, this suprathermal effect, which begins at amplitudes far below the level where non-linear hydrodynamic effects, of higher order in the fluid velocity, would arise [5, 7], causes a huge increase in the corresponding rates, effectively bringing them up to their ungapped level.

Beta equilibration processes have a finite time scale, so a harmonic local density fluctuation with amplitude  $\Delta n$  around the equilibrium value  $\bar{n}$  induces an oscillation in the displacement  $\mu_\Delta$  of the chemical potentials of the degenerate particles from their equilibrium. This oscillation leads both to heat generation (via dissipation due to bulk viscosity) and heat loss (from enhanced neutrino emission). Which of the two effects dominates depends among other factors on the amplitude of the oscillation

[6]. In [7] we found that over the entire range of physically reasonable density amplitudes the chemical potential oscillation is given by the linear, harmonic relation

$$\mu_\Delta(t) = \hat{\mu}_\Delta \sin(\omega t), \quad \hat{\mu}_\Delta = C \frac{\Delta n}{\bar{n}}, \quad C \equiv \bar{n} \left. \frac{\partial \mu_\Delta}{\partial n} \right|_x \quad (1)$$

where  $C$  is a susceptibility characterizing the particular form of dense matter.

We are interested in the direct ‘‘Urca’’ processes  $d \rightarrow u + l + \bar{\nu}_l$  and  $u + l \rightarrow d + \nu_l$  mediated by charged current interactions, which occur in hadronic matter at high densities where proton fraction  $x \gtrsim 10\%$  [16]. To study all relevant cases within a unified framework we use a notation where  $d$  represents either an elementary negatively charged quark (down or strange) or a hadron that contains that quark. Similarly  $u$  stands either for an elementary up quark or a hadron containing it;  $l$  represents a charged lepton (an electron, or in dense neutron matter a muon) and  $\nu_l$  is the corresponding neutrino. We specialize to Fermi liquid theory, where the general dispersion relation of the matter fields is

$$(E_i - \mu_i)^2 = v_{Fi}^2 (p_i - p_{Fi})^2 + \Delta_i^2 \quad (2)$$

where  $i$  labels the matter species and  $\Delta_i$  represents a gap in the particle spectrum arising from superfluidity or (color)-superconductivity. Non-Fermi liquid effects should not play a role in a gapped system [17]. We neglect dependence of the gap on energy and momentum, but we include temperature and density dependence  $\Delta_i = \Delta_i(T, \mu)$ . The leptons can to very good approximation be described by a free dispersion relation  $E_l^2 = p_l^2 + m_l^2$ ,  $E_\nu = p_\nu$ . Beta equilibrium with respect to the Urca processes enforces  $\mu_\Delta \equiv \mu_d - \mu_u - \mu_l = 0$ . Since compact stars contain ultradegenerate matter with  $T \ll \mu$  we can restrict the analysis to leading order in  $T/\mu$  where we find the general results for the net rate and the emissivity

$$\Gamma_{dU}^{(\leftrightarrow)} \approx \frac{17G^2}{120\pi} D \Theta T^4 \mu_\Delta R_\Gamma \left( \frac{\mu_\Delta}{T}, \frac{\Delta_d}{T}, \frac{\Delta_u}{T} \right) \quad (3)$$

$$\epsilon_{dU} \approx \frac{457\pi G^2}{5040} D \Theta T^6 R_\epsilon \left( \frac{\mu_\Delta}{T}, \frac{\Delta_d}{T}, \frac{\Delta_u}{T} \right) \quad (4)$$

In these expressions  $G$  is the effective coupling of the fields to the weak current,  $D$  is a function depending on the density and the equation of state

$$D = \frac{p_{Fd} p_{Fu}}{\mu_u v_{Fd} v_{Fu}} (\mu_d^2 - p_{Fd}^2 - \mu_u^2 + p_{Fu}^2 - m_l^2) \quad (5)$$

which is identical for both quantities, and the parenthesis vanishes for a free massless dispersion relation. Aside from the neglected temperature corrections any modification of the dispersion relations due to interactions

or masses opens phase space and yields a non-zero result. Eqs. (3) and (4) also contain a threshold function  $\Theta \approx \theta(p_{Fu} + p_{Fl} - p_{Fd})$ , a characteristic temperature and amplitude dependence, and a *modification function*  $R$  given for singlet gaps by the dimensionless integrals

$$\begin{aligned} R_i \left( \frac{\mu_\Delta}{T}, \frac{\Delta_d}{T}, \frac{\Delta_u}{T} \right) &= N_i \int_0^\infty dx_\nu x_\nu^{2+\lambda_i} \\ &\cdot \left( \int_{-\infty}^{-\frac{\Delta_d}{T}} + \int_{\frac{\Delta_d}{T}}^\infty \right) \frac{dx_d |x_d|}{\sqrt{x_d^2 - \frac{\Delta_d^2}{T^2}}} \left( \int_{-\infty}^{-\frac{\Delta_u}{T}} + \int_{\frac{\Delta_u}{T}}^\infty \right) \frac{dx_u |x_u|}{\sqrt{x_u^2 - \frac{\Delta_u^2}{T^2}}} \\ &\cdot \tilde{n}(x_d) \tilde{n}(-x_u) \left( \tilde{n} \left( x_\nu + x_u - x_d - \frac{\mu_\Delta}{T} \right) \right. \\ &\quad \left. - (-1)^{\lambda_i} \tilde{n} \left( x_\nu + x_u - x_d + \frac{\mu_\Delta}{T} \right) \right) \end{aligned} \quad (6)$$

where  $N_\Gamma = \frac{60}{17\pi^4}$ ,  $N_\epsilon = \frac{2520}{457\pi^6}$ ,  $\lambda_\Gamma = 0$  and  $\lambda_\epsilon = 1$ . These functions reflect the phase space available to Urca reactions. They are normalized so that  $R(0, 0, 0) = 1$  and are symmetric in the gap parameters  $R(x, y, z) = R(x, z, y)$ . In equilibrium they are pure reduction factors  $R(0, \dots) \leq 1$  that can become extremely small [1]. However, out of equilibrium  $\mu_\Delta \neq 0$  these modification functions exhibit a suprathermal enhancement for  $\mu_\Delta/T > 1$  [5–7] and can then greatly exceed one. They depend on the equation of state only via dimensionless ratios and are the same for all direct Urca processes [1]. In the ungapped case they have an analytic polynomial form [6].

The oscillation period of a compact star is much smaller than its evolution time scale, so the relevant quantity is the averaged emissivity  $\bar{\epsilon}_{dU} \equiv \frac{1}{2\pi} \int_0^{2\pi} d\varphi \epsilon_{dU}(\mu_\Delta(\varphi))$  which takes the same form eq. (4) with the averaged modification function  $R_\epsilon$  depending on  $\hat{\mu}_\Delta$ . The bulk viscosity of dense matter is [7]

$$\zeta = -\frac{C}{\pi \omega^2} \frac{\bar{n}_*}{\Delta n_*} \int_0^{2\pi} d\varphi \cos(\varphi) \int_0^\varphi d\varphi' \Gamma^{(\leftrightarrow)}(\mu_\Delta(\varphi')) \quad (7)$$

Inserting the rate eq. (3) and the low-amplitude form of the chemical potential oscillation eq. (1) gives

$$\zeta_{dU} = \frac{17G^2}{120\pi} C^2 D \Theta \frac{T^4}{\omega^2} R_\zeta \left( \frac{\hat{\mu}_\Delta}{T}, \frac{\Delta_d}{T}, \frac{\Delta_u}{T} \right) \quad (8)$$

with an analogous modification function

$$\begin{aligned} R_\zeta \left( \frac{\hat{\mu}_\Delta}{T}, \frac{\Delta_d}{T}, \frac{\Delta_u}{T} \right) &= -\frac{1}{\pi} \int_0^{2\pi} d\varphi \cos(\varphi) \int_0^\varphi d\varphi' \sin(\varphi') \\ &\cdot R_\Gamma \left( \frac{\hat{\mu}_\Delta}{T} \sin(\varphi'), \frac{\Delta_d}{T}, \frac{\Delta_u}{T} \right) \end{aligned} \quad (9)$$

In hyperon and quark matter the viscosity is dominated by neutral current rather than Urca processes, but for those we also expect a high-amplitude enhancement [7].

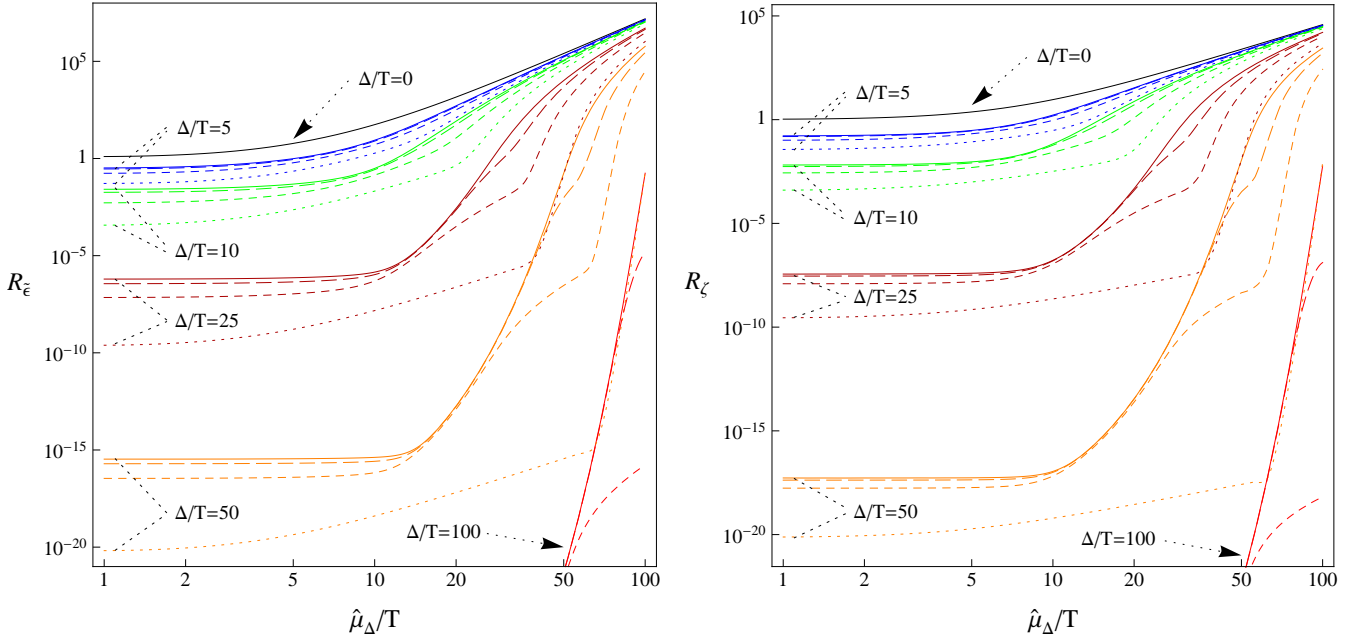


Figure 2: The modification functions  $R_i$  eqs. (4) & (8) as a function of oscillation amplitude  $\hat{\mu}_\Delta$ , for different pairing patterns with maximum gap  $\Delta$ : symmetric case where only one particle is gapped, e.g.  $\Delta_d = \Delta$ ,  $\Delta_u = 0$  (solid lines); intermediate cases  $\Delta_d = 5\Delta_u = \Delta$  (long dashed lines);  $\Delta_d = 2\Delta_u = \Delta$  (short dashed lines); symmetric case  $\Delta_d = \Delta_u = \Delta$  (dotted lines). Gap ranges from  $\Delta/T = 0$  to  $\Delta/T = 100$ . *Left panel:*  $R_\epsilon$  for the averaged neutrino emissivity. *Right panel:*  $R_\zeta$  eq. (9) for the bulk viscosity.

process	$G$	$f_V$	$g_A$	$D$	$\mu_e$	$C$	$\tilde{\epsilon} \left[ \frac{\text{ergs}}{\text{cm}^3 \text{s}} \right]$	$\tilde{\zeta} \left[ \frac{\text{g}}{\text{cm s}} \right]$	$\nu_e$	$\nu_\zeta$
$n \rightarrow p \bar{\nu}_l$	$\frac{1}{2} \sqrt{f_V^2 + 3g_A^2} \cos \theta_C G_F$	1	1.23	$2m_n^* m_p^* \mu_e$	$4(1-2x)S$	$4(1-2x) \left( n \frac{\partial S}{\partial n} - \frac{S}{3} \right)$	$6.82 \cdot 10^{26}$	$7.86 \cdot 10^{22}$	$\frac{2}{3}$	2
$\Lambda \rightarrow p \bar{\nu}_l$	$\frac{1}{2} \sqrt{f_V^2 + 3g_A^2} \sin \theta_C G_F$	-1.23	0.89	$2m_\Lambda^* m_p^* \mu_e$	or $\frac{(3\pi^2 n)^{2/3}}{2m_n}$	or $\frac{(3\pi^2 n)^{2/3}}{6m_n}$	$3.05 \cdot 10^{25}$	$3.52 \cdot 10^{21}$		
$\Sigma^- \rightarrow n \bar{\nu}_l$	$\frac{1}{2} \sqrt{f_V^2 + 3g_A^2} \sin \theta_C G_F$	-1	0.28	$2m_\Sigma^* m_n^* \mu_e$	for a free gas	for a free gas	$1.04 \cdot 10^{25}$	$1.20 \cdot 10^{21}$		
$d \rightarrow u e \bar{\nu}_e$	$\sqrt{3} \cos \theta_C G_F$	-	-	$\frac{8\alpha_s}{3\pi} \mu_q^2 \mu_e$	$\frac{m_s^2}{4\mu_q}$	$-\frac{m_s^2}{3(1-c)\mu_q}$	$1.57 \cdot 10^{25} \alpha_s$	$5.09 \cdot 10^{21} \alpha_s$	$\frac{1}{3}$	$-\frac{1}{3}$
$s \rightarrow u e \bar{\nu}_e$	$\sqrt{3} \sin \theta_C G_F$	-	-	$m_s^2 \mu_q$		free gas: $c = 0$	$3.98 \cdot 10^{24}$	$1.29 \cdot 10^{21}$		

Table I: Parameters for direct Urca processes in dense matter, with Fermi constant  $G_F$ , Cabbibo angle  $\theta_C$ , vector and axial couplings  $f_V$  and  $g_A$  [1], proton fraction  $x$ , symmetry energy  $S(\bar{n})$  [16], in-medium hadron masses  $m^*(\bar{n})$  and quark interaction parameter  $c$  [7, 18]. The quark expression is to leading order in  $\alpha_s$  and all results to leading order in  $T/\mu$ ,  $m_l/\mu$  and  $\mu_\Delta/\mu_e$ .

To see the gap-bridging enhancement of beta-equilibration rates for oscillations of suprathermal amplitude, without specifying a particular form of matter, we show in fig. 2 the modification functions  $R_\epsilon$  and  $R_\zeta$ . We vary the maximum gap  $\Delta$  and the ratio of the two gaps. For strongly gapped particles  $\Delta_i/T \gg 1$ , the larger gap tends to dominate, e.g.  $(\Delta_d, \Delta_u) = (2\Delta, 0)$  is more suppressed than  $(\Delta, \Delta)$ . Once the amplitude  $\hat{\mu}_\Delta$  becomes comparable to  $\Delta$ , the Urca reactions can bridge the gap(s) so that the modification functions rise steeply, and quickly reach their ungapped levels. Interestingly, for matter where the two gaps are the same (dotted lines) this rise starts already for  $\mu_\Delta \ll \Delta_i$  and eventually merges into the solid curve for a single gap  $\Delta_d + \Delta_u$ . In contrast, in the asymmetric case where one gap is much larger than the other, the effects of the smaller gap are only visible at large amplitudes and become negligible when it is more than an order of magnitude smaller.

Fig. 2 shows that suprathermal enhancement in gapped matter is considerably bigger than in ungapped matter (the  $\Delta/T = 0$  line) [7]. This enhancement could be realistically achieved in compact star oscillations:  $\hat{\mu}_\Delta$  is related to the density amplitude  $\Delta n/\bar{n}$  by the susceptibility  $C$  eq. (1). For hadronic matter with an APR equation of state [19],  $C$  increases from  $C(n_0/4) \approx 20$  MeV to  $C(5n_0) \approx 150$  MeV [2]; for quark matter it decreases from  $C(n_0) \approx 30$  MeV to  $C(5n_0) \approx 20$  MeV. Thus for amplitudes  $\Delta n/\bar{n} \sim 0.01$  the amplitude  $\hat{\mu}_\Delta$  can indeed become large enough to bridge gaps  $\Delta \sim 1$  MeV [7].

To apply these general results to explicit processes requires dispersion relations and effective couplings for the relevant particles. Here we consider direct Urca processes in hadronic [2, 16, 20], hyperonic [2, 21] and quark matter [22, 23]. Parameter values are given in tab. I. Inserted in the general results eqs. (4) and (8) they reproduce previous expressions in the subthermal case  $\mu_\Delta \ll T$ . In the

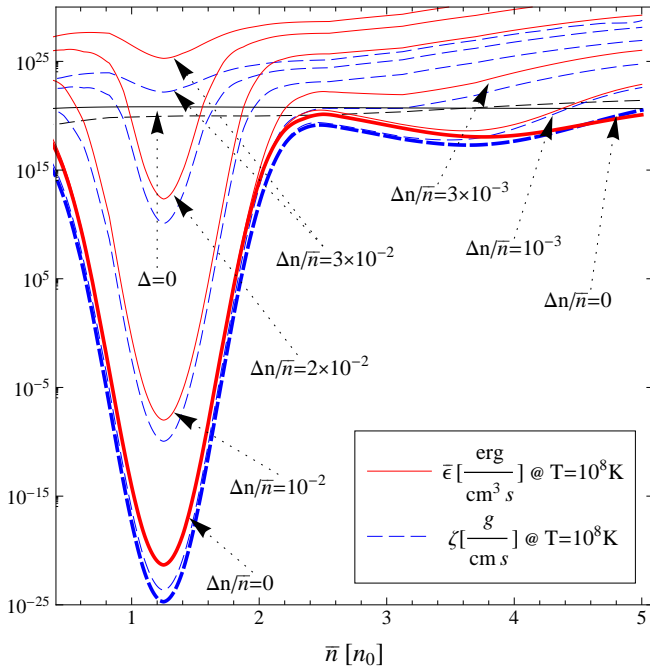


Figure 3: Enhancement via suprathermal oscillations of the neutrino emissivity and the bulk viscosity of hadronic matter with a  $^1S_0$  proton gap  $\Delta_{p0} = 1$  MeV at  $n \approx 1.3 n_0$  and a neutron gap  $\Delta_{n0} \approx 0.1$  MeV at  $n \approx 4 n_0$ .

ideal gas approximation our results simplify to

$$\bar{\epsilon} = \bar{\epsilon} \left( \frac{\bar{n}}{n_0} \right)^{\nu_{\bar{\epsilon}}} T_9^6 R_{\bar{\epsilon}}, \quad \bar{\zeta} = \bar{\zeta} \left( \frac{\bar{n}}{n_0} \right)^{\nu_{\bar{\zeta}}} \left( \frac{1 \text{ kHz}}{\omega} \right)^2 T_9^4 R_{\bar{\zeta}} \quad (10)$$

where  $T_9$  is the temperature in units of  $10^9$  K and the values of  $\bar{\epsilon}$  and  $\bar{\zeta}$  are given in tab. I.

As an explicit example that takes into account the complications in gapped interacting matter we consider APR hadronic matter [19] with the in-medium hadron masses given in [24]. For the density dependence of the  $^1S_0$  proton gap we use a Gaussian fit to data from [3], with maximum  $\Delta_{p0} = 1$  MeV centered at  $n \approx 1.3 n_0$ . For illustrative purposes we assume the neutrons also have a  $^1S_0$  gap (in reality it is expected to be  $^3P_2$ ) with the density dependence proposed to explain the Cas A cooling data in [14], i.e. with maximum  $\Delta_{n0} = 0.12$  MeV at  $n \approx 3.7 n_0$ . The gaps have a BCS temperature dependence, but this has only a modest effect in small regions where  $T \sim \Delta_p, \Delta_n$ . Our results for neutrino emissivity and bulk viscosity at  $T = 10^8$  K are shown in fig. 3. At infinitesimal amplitudes (thick  $\Delta n / \bar{n} = 0$  lines) there is enormous suppression by the proton gap, and moderate suppression by the smaller neutron gap. As the density oscillation amplitude increases, it first, at  $\Delta n / \bar{n} \gtrsim 10^{-3}$ , overcomes the suppression of emissivity and viscosity due to the neutron gap, increasing them at densities potentially relevant for direct Urca processes by many orders of magnitude. Then, at  $\Delta n / \bar{n} \gtrsim 10^{-2}$ , even the large proton

gap at lower density is bridged, boosting the results by up to  $10^{50}$ ! As in previous results for ungapped matter [7], these effects are expected to be even more pronounced for the realistic case of modified Urca reactions [1, 6], that are actually present at moderate densities.

Fig. 3 shows that, if the star is cold enough that Urca processes are blocked by pairing gaps throughout the star, density oscillations could provide the dominant contribution to those processes, by bridging the pairing gaps, starting in the regions where the rate-controlling gap is the smallest. The smaller this minimum gap, the lower the amplitude required to bridge it. In hyperonic [2, 21] or quark matter [25, 26] there are processes which are only suppressed by  $\Delta \lesssim 0.01$  MeV, that could be bridged by oscillations with amplitude as small as  $\Delta n / \bar{n} \lesssim 10^{-4}$ .

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